

Supplemental Material for Primitive Copy Sectors and Diffusive Infrared Closure

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1 Physical observable nets

The fragmentation theorem in the main text requires a physically accessible observable net. Let

$$\mathcal{H}_x = \bigoplus_{m \in M} \mathcal{K}_{x,m}. \quad (1)$$

The full microscopic algebra $\text{End}(\mathcal{H}_x)$ contains inter-sector matrix units $|m\rangle\langle m'|$; the observable net below retains the block-diagonal sector accessible to physical measurements. The observable net used in the theorem is

$$\mathfrak{A}_X^{\text{obs}} = \bigoplus_{m_X \in M^X} \text{End} \left(\bigotimes_{x \in X} \mathcal{K}_{x,m_x} \right). \quad (2)$$

Its centre contains the cylinder projections P_C for label events. This is the algebra used when the labels are superselected, gauge-hidden, or operationally inaccessible.

Lemma 1 (Commutant of the block observable net). *Let P_C be a finite label-cylinder projection. If $A \in \mathfrak{A}_X^{\text{obs}}$ is block diagonal in the label decomposition, then $[A, P_C] = 0$ whenever C is measurable with respect to the labels in X or disjoint from X after extension by identity. If the dynamics is label-preserving, then $[\alpha^n(A), P_C] = 0$ for all n whenever P_C is invariant under the induced label map.*

Proof. Both A and P_C are diagonal with respect to the same direct-sum decomposition over label configurations. Label preservation implies $\alpha^n(P_C) = P_C$ and carries block-diagonal observables to block-diagonal observables. The claim follows by direct multiplication on each sector. $\square$

2 Thermodynamic Kubo-Mori fluctuation space

Let ω be a faithful, translation-invariant state on a quasi-local C^* -algebra $\mathfrak{A}$ with modular group σ_t^ω in the GNS representation. For local zero-mean A, B define the connected Kubo-Mori covariance

$$C_\omega(A, B) = \int_0^1 [\omega(A^* \sigma_{-is}^\omega(B)) - \omega(A^*)\omega(B)] ds. \quad (3)$$

Assume exponential clustering: there are $C, \mu > 0$ such that

$$|C_\omega(A, \tau_x(B))| \leq C \|A\| \|B\| e^{-\mu \text{dist}(\text{supp } A, \text{supp } \tau_x B)} \quad (4)$$

for all local A, B . Then

$$s_\omega(A, B) = \sum_{x \in \mathbb{Z}^d} C_\omega(A, \tau_x(B)) \quad (5)$$

converges absolutely and is positive semidefinite.

Let $\mathcal{N} = \{A : s_\omega(A, A) = 0\}$. The fluctuation Hilbert space is

$$\mathcal{H}_\text{fl} = \overline{\mathfrak{A}_{\text{loc}}^0 / \mathcal{N}^{s_\omega}}. \quad (6)$$

Equivalently, for smooth compactly supported test functions f and boxes Λ_L ,

$$F_L(f, A) = L^{-d/2} \sum_{x \in \Lambda_L} f(x/L) (\tau_x(A) - \omega(A)\mathbf{1}) \quad (7)$$

has covariance

$$\lim_{L \rightarrow \infty} \omega(F_L(f, A)^* F_L(g, B)) = \left(\int_{\mathbb{R}^d} \bar{f}(u) g(u) du \right) s_\omega(A, B) \quad (8)$$

under the same clustering hypothesis.

3 Closed forms and sectoriality

Let $\mathcal{C} \subset \mathcal{H}_\text{fl}$ be the image of local observables and let $\mathfrak{e}$ be a densely defined sesquilinear form on $\mathcal{C}$. It is sectorial with vertex zero if

$$\operatorname{Re} \mathfrak{e}[u, u] \geq 0, \quad |\operatorname{Im} \mathfrak{e}[u, u]| \leq \tan \theta \operatorname{Re} \mathfrak{e}[u, u] \quad (9)$$

for some $\theta < \pi/2$. It is coercive on $Q\mathcal{H}_\text{fl}$ if

$$\operatorname{Re} \mathfrak{e}[u, u] \geq c \|u\|^2, \quad u \in Q\mathcal{C}. \quad (10)$$

If $\mathfrak{e}$ is closable, its closure generates a unique maximal sectorial operator M satisfying

$$\mathfrak{e}[u, v] = \langle u, Mv \rangle \quad (11)$$

for $v \in \operatorname{Dom}(M)$ and $u \in \operatorname{Dom}(\bar{\mathfrak{e}})$. Coercivity implies $0 \in \rho(M)$ and

$$\|M^{-1}\| \leq c^{-1} \sec \theta. \quad (12)$$

This is the analytic input used in the Feshbach-Schur proof.

4 Hydrodynamic fibres

Translation invariance diagonalizes fluctuation fields into Bloch fibres. For a finite box with periodic boundary conditions, write

$$A_k = |\Lambda_L|^{-1/2} \sum_{x \in \Lambda_L} e^{-ik \cdot x} \tau_x(A), \quad k \in (2\pi/L)\mathbb{Z}^d. \quad (13)$$

If the finite-volume generators L_L have a strong-resolvent limit on the fluctuation Hilbert space and commute with translations, then

$$L = \int_{\mathbb{T}^d}^\oplus L(k) dk. \quad (14)$$

The analytic hypothesis in the main theorem is a model-level condition. It is verified directly in the exchange-dephasing sector and can also be obtained from a controlled projection method. One sufficient condition is a common form core $\mathcal{C}$ and a relative expansion

$$\|(L(k) - L_0 - ik_i V_i + k_i k_j W_{ij})u\| \leq C|k|^3 \|(1 - L_0)u\|, \quad u \in \mathcal{C}. \quad (15)$$

5 Detailed Feshbach-Schur expansion

Let $P = \chi^{-1}|r\rangle\langle r|$ and $Q = 1 - P$. Let $M = -QL_0Q$. By the sectorial gap,

$$Q(L(k) - \lambda)Q = -M + Q\Delta(k, \lambda)Q, \quad (16)$$

where $\|\Delta(k, \lambda)M^{-1}\| < 1$ for $|k| + |\lambda|$ sufficiently small. Thus

$$[Q(L(k) - \lambda)Q]^{-1} = -M^{-1} - M^{-1}Q\Delta(k, \lambda)QM^{-1} + O((|k| + |\lambda|)^2). \quad (17)$$

Substituting into the Feshbach map gives

$$F(k, \lambda) = -\lambda P - k_i k_j P W_{ij} P - k_i k_j P V_i Q M^{-1} Q V_j P \quad (18)$$

$$+ O(|k|^3 + |k|^2|\lambda| + |\lambda|^2). \quad (19)$$

Since P is one-dimensional, $F(k, \lambda) = 0$ is a scalar equation. The implicit-function theorem gives the unique branch

$$\lambda(k) = -k_i k_j \chi^{-1} \left(\langle r, W_{ij} r \rangle + \langle Q V_i r, M^{-1} Q V_j r \rangle \right) + O(|k|^3). \quad (20)$$

This is the formula stated in the main text.

6 Exchange-dephasing model

On a finite torus let

$$\mathcal{L}(A) = \sum_{\langle x, y \rangle} \sum_{s=\pm} \left(v_{xy, s}^* A v_{xy, s} - \frac{1}{2} \{v_{xy, s}^*, v_{xy, s}, A\} \right) + \mathcal{L}_{\text{dep}}(A) \quad (21)$$

with the jump operators given in the main text. Since for every edge both orientations are present and $\omega_\rho = \bigotimes_x ((1 - \rho)|0\rangle\langle 0| + \rho|1\rangle\langle 1|)$ satisfies detailed balance for the exchange dynamics, $\mathcal{L}$ is self-adjoint in the Kubo-Mori form on the closed one-density fluctuation sector. The conserved density obeys

$$\mathcal{L}(n_x) = \gamma \sum_{i=1}^d (n_{x+e_i} + n_{x-e_i} - 2n_x). \quad (22)$$

Therefore the Fourier mode q_k satisfies

$$\mathcal{L}(q_k) = -2\gamma \sum_{i=1}^d (1 - \cos k_i) q_k. \quad (23)$$

The hydrodynamic tensor is $D_{ij} = \gamma \delta_{ij}$. The dephasing term does not change q_k but adds a strictly positive decay rate to coherences. On the closed one-density fluctuation sector generated by q_k , the only zero mode at $k = 0$ is the scalar charge density. Thus the primitive-centre and sectoriality assumptions are verified in this explicit sector.

7 Monotone obstruction under admissible RG

Let E be a state-preserving conditional expectation. Petz monotonicity gives

$$\langle E(A), E(A) \rangle_{\text{KM}, E_*(\omega)} \leq \langle A, A \rangle_{\text{KM}, \omega}. \quad (24)$$

Hence the squared norm of any projected current cannot increase. Let $\mathcal{Z}$ be the centre of the hydrodynamic algebra and $\mathcal{Z}_0$ the primitive centre. If q is a quotient by syndrome-trivial central projections, then

$$\dim_{\text{MvN}}(q\mathcal{Z}q/q\mathcal{Z}_0q) \leq \dim_{\text{MvN}}(\mathcal{Z}/\mathcal{Z}_0). \quad (25)$$

For sectorial forms, compression and quotient by null subspaces preserve membership in the closed sectorial cone. Finally, the metric measure condition in the definition of admissibility requires preservation of absolute continuity on retained sectors. These statements prove the termwise monotonicity of Φ .

8 Hypercharge algebra

The anomaly calculation used in the restricted gauge-code application is elementary but included for reproducibility. Yukawa closure gives

$$y_u = y_q + y_H, \quad y_d = y_q - y_H, \quad y_e = y_\ell - y_H. \quad (26)$$

The $SU(2)^2U(1)$ condition gives $y_\ell = -3y_q$. The gravitational anomaly then gives

$$6y_q - 3(y_q + y_H) - 3(y_q - y_H) + 2(-3y_q) - (-3y_q - y_H) = 0, \quad (27)$$

which reduces to $y_H = -3y_q - y_e$. With $y_e = -1$ and the cubic anomaly, the nontrivial chiral solution is

$$y_q = \frac{1}{6}, \quad y_H = \frac{1}{2}, \quad y_u = \frac{2}{3}, \quad y_d = -\frac{1}{3}, \quad y_\ell = -\frac{1}{2}, \quad y_e = -1. \quad (28)$$

Vectorlike pairs or sterile singlets are outside this one-generation non-spectator calculation unless separately assigned copy syndromes and morphisms.

9 Scalar threshold arithmetic

The Maxwell radial mean in three dimensions is

$$\mathbb{E}|Z| = \frac{1}{(2\pi)^{3/2}} 4\pi \int_0^\infty r^3 e^{-r^2/2} dr = \sqrt{\frac{8}{\pi}}. \quad (29)$$

The copy suppression required to reduce σ_{raw} below a limit σ_{lim} is

$$S_{\text{copy}} \geq \frac{1}{2} \log \left(\frac{\sigma_{\text{raw}}}{\sigma_{\text{lim}}} \right). \quad (30)$$

For $\sigma_{\text{raw}} = 3.8 \times 10^{-28} \text{ cm}^2$ and $\sigma_{\text{lim}} = 4.0 \times 10^{-48} \text{ cm}^2$ this gives

$$S_{\text{copy}} \geq 22.974 \dots \quad (31)$$

Rounding upward gives the stated threshold $S_{\text{copy}} \simeq 23.0$.

References

- [1] E. H. Lieb and D. W. Robinson, The finite group velocity of quantum spin systems, *Communications in Mathematical Physics* **28**, 251–257 (1972).
- [2] S. Bravyi, M. B. Hastings, and F. Verstraete, Lieb-Robinson bounds and the generation of correlations and topological quantum order, *Physical Review Letters* **97**, 050401 (2006).
- [3] D. Gross, V. Nesme, H. Vogts, and R. F. Werner, Index theory of one dimensional quantum walks and cellular automata, *Communications in Mathematical Physics* **310**, 419–454 (2012).
- [4] J. Haah and M. B. Hastings, Classification of quantum cellular automata, *Communications in Mathematical Physics* **376**, 1179–1222 (2020).
- [5] R. Kubo, Statistical-mechanical theory of irreversible processes. I, *Journal of the Physical Society of Japan* **12**, 570–586 (1957).
- [6] H. Mori, Transport, collective motion, and Brownian motion, *Progress of Theoretical Physics* **33**, 423–455 (1965).
- [7] H. Spohn, *Large Scale Dynamics of Interacting Particles*, Springer, Berlin (1991).
- [8] T. Kato, *Perturbation Theory for Linear Operators*, Springer, Berlin (1966).
- [9] O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 2*, 2nd ed., Springer, Berlin (1997).
- [10] H. Araki, Relative entropy of states of von Neumann algebras, *Publications of the Research Institute for Mathematical Sciences* **11**, 809–833 (1976).
- [11] D. Petz, Monotone metrics on matrix spaces, *Linear Algebra and its Applications* **244**, 81–96 (1996).
- [12] G. Lindblad, On the generators of quantum dynamical semigroups, *Communications in Mathematical Physics* **48**, 119–130 (1976).
- [13] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Completely positive dynamical semigroups of N-level systems, *Journal of Mathematical Physics* **17**, 821–825 (1976).
- [14] C. Kipnis and C. Landim, *Scaling Limits of Interacting Particle Systems*, Springer, Berlin (1999).
- [15] E. B. Davies, *Quantum Theory of Open Systems*, Academic Press, London (1976).
- [16] H. B. Nielsen and M. Ninomiya, Absence of neutrinos on a lattice. I. Proof by homotopy theory, *Nuclear Physics B* **185**, 20–40 (1981).
- [17] M. Luescher, Exact chiral symmetry on the lattice and the Ginsparg-Wilson relation, *Nuclear Physics B* **568**, 162–179 (2000).
- [18] D. S. Freed and M. J. Hopkins, Reflection positivity and invertible topological phases, *Geometry & Topology* **25**, 1165–1330 (2021).
- [19] S. Weinberg, A model of leptons, *Physical Review Letters* **19**, 1264–1266 (1967).
- [20] H. Georgi and S. L. Glashow, Unity of all elementary-particle forces, *Physical Review Letters* **32**, 438–441 (1974).

- [21] V. Silveira and A. Zee, Scalar phantoms, *Physics Letters B* **161**, 136–140 (1985).
- [22] J. McDonald, Gauge singlet scalars as cold dark matter, *Physical Review D* **50**, 3637–3649 (1994).
- [23] ATLAS Collaboration, Combination of searches for invisible decays of the Higgs boson using 139 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13\text{ TeV}$ collected with the ATLAS experiment, *Physics Letters B* **842**, 137963 (2023).
- [24] XENON Collaboration, WIMP dark matter search using a 3.1 tonne year exposure of the XENONnT experiment, *Physical Review Letters* **135**, 221003 (2025); arXiv:2502.18005.
- [25] LZ Collaboration, Dark matter search results from 4.2 tonne-years of exposure of the LUX-ZEPLIN (LZ) experiment, *Physical Review Letters* **135**, 011802 (2025); arXiv:2410.17036.